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LARGE DEFLECTION AND LARGE-AMPLITUDE FREE VIBRATIONS OF LAMINATED COMPOSITE-MATERIAL PLATES

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LARGE DEFLECTION AND LARGE-AMPLITUDE FREE VIBRATIONS OF LAMINATED COMPOSITE-MATERIAL PLATES

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Abstract - Finite-element analysis of the large-deflection theory (in von Karman's sense), including transverse shear, governing moderately thick, laminated anisotropic composite plates is presented. Linear and quadratic rectangular elements with five degrees of freedom (three displacements, and two shear rotations) per node are employed to analyze rectangular plates subjected to various loadings and edge conditions. Numerical results for bending deflections, stresses, and natural frequencies are presented showing the parametric effects of plate aspect ratio, side-to-thickness ratio, orientation of layers, and anisotropy.

1. INTRODUCTION

In the finite-element analysis of nonlinear problems the geometric stiffness matrix is reformulated several times during each load step (also, during each time step in the transient analysis), consequently, the computational time involved is very large. Further, if the element used in the analysis has many degrees of freedom, storage considerations may preclude the use of such elements. These concerns are reflected in current research in computational mechanics, which is largely concerned

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with the development of numerical schemes that are computationally inexpensive but possess competitive accuracy when compared to traditional schemes.

Due to their high stiffness-to-weight ratio, and the flexible anisotropic property that can be tailored through variation of the fiber orientation and stacking sequence, fiber-reinforced laminated composites are finding increasing application in many engineering structures. Plates are common in many engineering structures, and therefore have received greater attention of the designer.

Much of the previous research in the analysis of composite plates is limited to linear problems (see, for example, [1-15]), and many of them were based on the classical thin-plate theory (see [1-3]), which neglects the transverse shear deformation effects. The transverse shear effects are more pronounced, due to their low transverse shear modulus relative to the in-plane Young's moduli, in filamentary composite plates than in isotropic plates. The shear deformable theory of Yang, Norris, and Stavsky [16] (see also, Whitney and Pagano [17]), which is a generalization of Mindlin's theory for homogeneous, isotropic plates to arbitrarily laminated anisotropic plates, is now considered to be adequate for predicting the overall behavior such as transverse deflections and the first few natural frequencies of layered composite plates. Finite-element analysis of rectangular plates based on the Yang-Norris-Stavsky (YNS) theory is due to Reddy [15,18], who derived the YNS theory from the penalty function method of Courant [19]. A comparison of the closed-form solutions [17] with the finite-element solutions [14,15] shows that the element predicts accurate solutions (see also [20]).

Approximate solutions to the large-deflection theory (in von Karman's sense) of laminated composite plates were attempted by Whitney and Leissa [21], Bennett [22], Bert [23], Chandra and Raju [24,25], Zaghloul and Kennedy [26], Chia and Prabhakara [27,28], and Noor and Hartley [29]. Chandra and Raju [24,25], and Chia and Prabhakara [27,28] employed the Galerkin method to reduce the governing nonlinear partial differential equations to an ordinary differential equation in time for the mode shape; the perturbation technique was used to solve the resulting equation. Zaghloul and Kennedy [26] used a finite-difference successive iterative technique in their analysis. In all of these studies, the transverse shear effects were neglected. The finite element employed by Noor and Hartley [29] includes the effect of transverse shear strains; however, it is algebraically complex and involves eighty degrees of freedom per element. Use of such elements in the nonlinear analysis of composite plates inevitably leads to large storage requirements and computational costs.

The present paper is concerned with the large-deflection bending and large-amplitude free vibrations of laminated composite plates. The finite element used herein is a rectangular element based on the extended YNS theory (i.e., the transverse shear deformation is included) that includes the effect of large deflections (in the von Karman sense). The element has three displacements and two shear rotations per node and results in a 20 by 20 stiffness matrix for linear element and a 40 by 40 matrix for an eight-node quadratic element. Numerical results are presented for deflections, stresses, and natural frequencies of rectangular plates for various edge conditions.

2. GOVERNING EQUATIONS OF MODERATELY THICK PLATES ACCOUNTING FOR LARGE DEFLECTIONS

Consider a plate laminated of thin anisotropic layers, oriented arbitrarily, and having a total thickness h. The origin of the coordinate system (x,y) is taken in the middle plane, denoted R, of the plate with the z-axis perpendicular to the plane of the plate. The thick plate theory of Whitney and Pagano [17] is modified here to include the non-linear terms of the von Karman theory. The displacement field is assumed to be of the form,

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\,\psi_x(x,y,t) \;, \\ u_2(x,y,z,t) &= v(x,y,t) + z\,\psi_y(x,y,t) \;, \\ u_3(x,y,z,t) &= w(x,y,t) \;. \end{aligned} \tag{2.1}$$

Here t is the time; u_1, u_2, u_3 are the displacements in x, y, z directions, respectively; u, v, w are the associated midplane displacements; and ψ_x and ψ_y are the slopes in the xz and yz planes due to bending only. Assuming that the plate is moderately thick and strains are much smaller than rotations, we write the nonlinear strain-displacement relations $2\varepsilon_{ij} = u_{i,j} + u_{j,i} + u_{m,i}u_{m,j}$,

$$\begin{aligned} \varepsilon_1 &\equiv \varepsilon_{11} = \frac{\partial u}{\partial x} + z\,\frac{\partial \psi_x}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = \varepsilon_{11}^0 + zK_1 \;, \\ \varepsilon_2 &\equiv \varepsilon_{22} = \frac{\partial v}{\partial y} + z\,\frac{\partial \psi_y}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \varepsilon_{22}^0 + zK_2 \;, \\ \varepsilon_6 &\equiv 2\varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \;, \\ &= \varepsilon_6^0 + zK_6 \,, \end{aligned}$$

$$\epsilon_3 \equiv \epsilon_{33} = \psi_x^2 + \psi_y^2 \; , \; \epsilon_5 = \psi_x + \frac{\partial w}{\partial x} \; , \; \epsilon_4 = \psi_y + \frac{\partial w}{\partial y} \; . \tag{2.2}$$

wherein the products of ψ_x and ψ_y with $\partial u_1/\partial x$ and $\partial u_2/\partial y$ are neglected. Since the constitutive relations are based on the plane-stress assumption, ϵ_3 does not enter the formulation.

Neglecting the body moments and surface shearing forces, one can write the equations of motion (in the absence of body forces) as

$$\begin{aligned}
N_{1,x} + N_{6,y} &= Ru_{,tt} + S\psi_{x,tt} \\
N_{6,x} + N_{2,y} &= Rv_{,tt} + S\psi_{y,tt} \\
Q_{1,x} + Q_{2,y} &= P + Rw_{,tt} - \mathcal{N}(N_i, w) \\
M_{1,x} + M_{6,y} - Q_1 &= I\psi_{x,tt} + Su_{,tt} \\
M_{6,x} + M_{2,y} - Q_2 &= I\psi_{y,tt} + Sv_{,tt}
\end{aligned} \tag{2.3}$$

where R, S, and I are the normal, coupled normal-rotary, and rotary inertia coefficients,

$$(R,S,I) = \int_{-h/2}^{h/2} (1,z,z^2)\,\rho dz = \sum_m \int_{z_m}^{z_{m+1}} (1,z,z^2)\,\rho^{(m)}\,dz \tag{2.4}$$

$\rho^{(m)}$ being the material density of the m-th layer, P is the transversely distributed force, and N_i, Q_i, and M_i are the stress and moment resultants defined by

$$(N_i,M_i) = \int_{-h/2}^{h/2} (1,z)\,\sigma_i\,dz \; , \; (Q_1,Q_2) = \int_{-h/2}^{h/2} (\sigma_{xz},\sigma_{yz})\,dz \tag{2.5}$$

Here σ_i ($i = 1,2,6$) denote the in-plane stress components ($\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_4 = \sigma_{yz}$, $\sigma_5 = \sigma_{xz}$ and $\sigma_6 = \sigma_{xy}$). The nonlinear operator $\mathcal{N}(\cdot)$ in eqn. (2.3) is given by,

$$\mathcal{N}(w,N_i) = \frac{\partial}{\partial x}\left(N_1 \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(N_6 \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial x}\left(N_6 \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_2 \frac{\partial w}{\partial y}\right)$$

Assuming monoclinic behavior (i.e., existence of one plane of elastic symmetry parallel to the plane of the layer) for each layer, the constitutive equations for the m-th layer (in the plate coordinates) are given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = [Q_{ij}^{(m)}] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} , \begin{Bmatrix} \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} Q_{44}^{(m)} & Q_{45}^{(m)} \\ Q_{45}^{(m)} & Q_{55}^{(m)} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} , \quad (2.6)$$

where $Q_{ij}^{(m)}$ are the stiffness coefficients of the m-th layer in the plate coordinates. Combining eqns. (2.5) and (2.6), we obtain the plate constitutive equations,

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ji} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^o \\ K_j \end{Bmatrix} , \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} k_4^2\bar{A}_{44} & k_4k_5\bar{A}_{45} \\ k_4k_5\bar{A}_{45} & k_5^2\bar{A}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} . \quad (2.7)$$

The A_{ij}, B_{ij}, D_{ij} $(i,j = 1,2,6)$, and $\bar{A}_{ij}$ $(i,j = 4,5)$ are the respective inplane, bending-inplane coupling, bending or twisting, and thickness-shear stiffnesses, respectively:

$$(A_{ij},B_{ij},D_{ij}) = \sum_m \int_{z_m}^{z_{m+1}} Q_{ij}^{(m)} (1,z,z^2)dz \; , \; \bar{A}_{ij} = \sum_m \int_{z_m}^{z_{m+1}} Q_{ij}^{(m)} \, dz. \quad (2.8)$$

Here z_m denotes the distance from the mid-plane to the lower surface of the m-th layer.

Equations (2.3) and (2.7) must be adjoined by appropriate boundary conditions of the problem. The variational formulation of these equations indicate the following essential and natural boundary conditions:

$$\begin{aligned} &\underline{\text{essential}}: \text{ specify, } u_n,\ u_s,\ w,\ \psi_n, \\ &\underline{\text{natural}}: \text{ specify, } N_n,\ N_{ns},\ q,\ M_n,\ M_{ns}. \end{aligned} \quad (2.9)$$

3. VARIATIONAL FORMULATION

Toward constructing a finite-element model of eqns. (2.3), (2.7), and (2.9), we present a (quasi-) variational formulation of these equations. The total potential energy principle for the problem at hand takes the form,

$$
\begin{aligned}
0 = \delta\pi(u,v,w,\psi_x,\psi_y) = \int_R & \{\delta u[Ru_{,tt} + S\psi_{x,tt} - N_{1,x} - N_{6,y}] \\
& + \delta v[Rv_{,tt} + S\psi_{y,tt} - N_{6,x} - N_{2,y}] \\
& + \delta w[P + Rw_{,tt} - Q_{1,x} - Q_{2,y} - \mathcal{N}(w,N_i)] \\
& + \delta\psi_x[I\psi_{x,tt} + Su_{,tt} - M_{1,x} - M_{6,y} + Q_1] \\
& + \delta\psi_y[I\psi_{y,tt} + Sv_{,tt} - M_{6,x} - M_{2,y} + Q_2]\}\, dxdy, \\
= \int_R & \{\delta u(Ru_{,tt} + S\psi_{x,tt}) + \delta u_{,x}N_1 + \delta u_{,y}N_6 + \delta v(Rv_{,tt} + S\psi_{y,tt}) \\
& + \delta v_{,x}N_6 + \delta v_{,y}N_2 + \delta w(P + Rw_{,tt}) + \delta w_{,x}Q_1 + \delta w_{,y}Q_2 \\
& + \frac{\partial\delta w}{\partial x}\frac{\partial w}{\partial x}N_1 + \frac{\partial\delta w}{\partial y}\frac{\partial w}{\partial x}N_6 + \frac{\partial\delta w}{\partial x}\frac{\partial w}{\partial y}N_6 + \frac{\partial\delta w}{\partial y}\frac{\partial w}{\partial y}N_2 \\
& + \delta\psi_x(I\psi_{x,tt} + Su_{,tt}) + \delta\psi_{x,x}M_1 + \delta\psi_{x,y}M_6 + \delta\psi_x Q_1 \\
& + \delta\psi_y(I\psi_{y,tt} + Sv_{,tt}) + \delta\psi_{y,x}M_6 + \delta\psi_{y,y}M_2 + \delta\psi_y Q_2\}\, dxdy \\
& + \int_{C_n}(\delta u_n\hat{N}_n + \delta u_s\hat{N}_{ns})ds + \int_{C_q}\delta w\hat{q}ds + \int_{C_m}(\delta\psi_n\hat{M}_n + \delta\psi_s\hat{M}_{ns})ds,
\end{aligned}
\tag{3.1}
$$

wherein quantitites with '$\wedge$' are specified on the respective portions of the boundary C, and C_n, C_q and C_m are respectively the (possibly overlapping) portions of the boundary on which $\hat{N}_n$ and $\hat{N}_{ns}$, $\hat{q}$, and $\hat{M}_n$ and $\hat{M}_{ns}$ are specified. It should be noted that on the complements of these portions (i.e., on $C-C_n$, $C-C_q$, and $C-C_m$) the in-plane displacements u_n, and u_{ns}, transverse deflection w, and shear rotations ψ_n and ψ_{ns}, respectively, are specified.

4. FINITE-ELEMENT MODEL

Now we present a finite-element model based on the variational form in Eqn. (3.1). Suppose that the region R is divided into a finite number of rectangular elements. Over each element the generalized displacements $(u, v, w, \psi_x, \psi_y)$ are interpolated by

$$u = \sum_i^r u_i \phi_i^1 \ , \ v = \sum_i^r v_i \phi_i^1 \ , \ w = \sum_i^s w_i \phi_i^2 \ ,$$
$$\psi_x = \sum_i^p \psi_{xi} \phi_i^3 \ , \ \psi_y = \sum_i^p \psi_{yi} \phi_i^3 \ , \tag{4.1}$$

where ϕ_i^α $(\alpha = 1,2,3)$ is the interpolation function corresponding to the i-th node in the element. Note that the in-plane displacements, the transverse displacement, and the slope functions are approximated by different sets of interpolation functions. While this generality is included in the formulation (to indicate the fact that such independent approximations are possible), we dispense with it in the interest of simplicity when the element is actually programmed and take $\phi_i^1 = \phi_i^2 = \phi_i^3$ $(r = s = p)$. Here r, s, and p denote the number of degrees of freedom per each variable. That is, the total number of degrees of freedom per element is $2r + s + 2p$.

Substituting eqn. (4.1) into eqn. (3.1), we obtain

$$[K]\{\Delta\} = \omega^2 \ [M]\{\Delta\} + \{F\} \tag{4.2}$$

For static bending, eqn. (4.2) becomes

$$\begin{bmatrix} [K^{11}] & [K^{12}] & [0] & [K^{14}] & [K^{15}] \\ [K^{12}] & [K^{22}] & [0] & [K^{24}] & [K^{25}] \\ [0] & [0] & [K^{33}] & [K^{34}] & [K^{35}] \\ [K^{14}] & [K^{24}] & [K^{34}] & [K^{44}] & [K^{45}] \\ [K^{15}] & [K^{25}] & [K^{35}] & [K^{45}] & [K^{55}] \end{bmatrix}_e \left| \begin{matrix} \{u\} \\ \{v\} \\ \{w\} \\ \{\psi_x\} \\ \{\psi_y\} \end{matrix} \right|_e = \left\{ \begin{matrix} \{F^1\} \\ \{F^2\} \\ \{F^3\} \\ \{F^4\} \\ \{F^5\} \end{matrix} \right\}_e , \tag{4.3}$$

where the {u}, {v}, etc. denote the columns of the nodal values of u, v, etc. respectively, and the elements $K_{ij}^{\alpha\beta}$ ($\alpha,\beta = 1,2,\ldots,5$) of the stiffness matrix and F_i^α of the force vector can be identified easily from eqn.(3.1).

In the present study rectangular elements with four, eight, and nine nodes are employed with the same interpolation for all of the variables. The resulting stiffness matrices are 20 by 20 for the 4-node element and 40 by 40 for the 8-node element.

As pointed out in a recent study [15], the YNS theory can be derived from the corresponding classical thin-plate theory by treating the slope-displacement relations

$$\frac{\partial w}{\partial x} = -\theta_x \, , \; \frac{\partial w}{\partial y} = -\theta_y \, , \tag{4.4}$$

as constraints. Indeed, when the constraints in eqn. (4.4) are incorporated into the classical thin-plate theory by means of the penalty-function method, the resulting equations correspond to the YNS theory with the correspondence,

$$\theta_x \sim \psi_x \, , \; \theta_y \sim \psi_y \, . \tag{4.5}$$

It is now well-known that whenever the penalty-function method is used, the so-called reduced integration (see Zienkiewicz et al. [30], and Reddy [20]) must be used to evaluate the matrix coefficients in eqn. (4.3). That is, if the four-node rectangular element is used, the 1 x 1 Gauss rule must be used in place of the standard 2 x 2 Gauss rule to numerically evaluate the coefficients K_{ij}. The element equations in (4.2) are assembled in the usual manner, and the (essential) boundary conditions are imposed before solving either for generalized displacements, or for frequencies of natural vibration. It should be noted that, since the stiffness matrix [K] depends on the solution {Δ}, any one of the standard iterative procedures must be used.

5. NUMERICAL RESULTS AND DISCUSSION

The finite element presented herein was employed in the nonlinear analysis of rectangular plates. The following material properties typical of advanced fiber-reinforced composites were used in the present study:

$$\begin{aligned}&\text{Material I:} \quad E_1/E_2 = 25,\ G_{12}/E_2 = 0.5,\ G_{23}/E_2 = 0.2,\ \nu_{12} = 0.25\\ &\text{Material II:} \quad E_1/E_2 = 40,\ G_{12}/E_2 = 0.6,\ G_{23}/E_2 = 0.5,\ \nu_{12} = 0.25\end{aligned} \tag{5.1}$$

It was assumed that $G_{13} = G_{23}$ and $\nu_{12} = \nu_{13}$. A value of 5/6 was used for the shear correction coefficients, $k_4^2 = k_5^2$ (see Whitney [31]). All of the computations were carried on an IBM 370/158 computer.

To show the effect of the reduced integration, and to illustrate the accuracy of the present element, results of the linear analysis are presented for four-layer (equal thickness) cross-ply (0°/90°/90°/0°) square plate constructed of material I. The plate is subjected to sinusoidal distribution of transverse loading, and is assumed to be "simply-supported" in the following sense (SS-1):

$$\begin{aligned}&u_o(x,0) = u_o(x,b) = 0,\ N_2(x,0) = N_2(x,b) = 0 \ ,\\ &v_o(0,y) = v_o(a,y) = 0,\ N_1(0,y) = N_1(a,y) = 0 \ ,\\ &w(x,0) = w(x,b) = w(0,y) = w(a,y) = 0 \ ,\\ &\psi_x(x,0) = \psi_x(x,b) = 0,\ M_2(x,0) = M_2(x,b) = 0 \ ,\\ &\psi_y(0,y) = \psi_y(a,y) = 0,\ M_1(0,y) = M_1(a,y) = 0 \ .\end{aligned} \tag{5.2}$$

Of course, in the finite-element method only the essential boundary conditions (i.e., those on u, v, w, ψ_x and ψ_y) are imposed after the assembly of element equations. The finite-element solution is compared with the closed-form solution [20], and the 3-D elasticity solution of Pagano and

Hatfield [5] in Figure 1. It is clear from the figure that nondimensionalized deflection obtained by 2 by 2 mesh of linear elements is very sensitive to the integration (i.e., reduced and full integration) in the thin-plate range (i.e., $a/h > 20$). However, the integration has virtually no noticeable effect in the thick-plate range, or for quadratic elements. The solutions obtained using the quadratic elements (with reduced and full integration) are not plotted in Figure 1 due to their closeness to the closed-form solution. The solutions (i.e., deflections and stresses) obtained by various elements, meshes, and integrations are reported in tabular form in [20]. The solution obtained by the 4 by 4 mesh of quadratic elements is in excellent agreement (indeed, to the third decimal point) with the closed-form solution. The stresses $\bar{\sigma}_x$ and $\bar{\sigma}_y$ were computed at the Gauss point $x = y = 0.0625$ (close to the center of the plate) at $z = \pm h/2$, and $\pm h/4$, respectively.

Figure 1 also shows the nondimensionalized deflection for four-layer, angle-ply (45°/-45°/45°/-45°) square plate (material II) under sinusoidal loading. The boundary conditions used are of simply-supported (SS-2) type:

$$\begin{aligned}
&u_0(0,y) = u_0(a,y) = 0 \;,\; N_6(0,y) = N_6(a,y) = 0 \;, \\
&v_0(x,0) = v_0(x,b) = 0 \;,\; N_6(x,0) = N_6(x,b) = 0 \;, \\
&w(x,0 = w(x,b) = w(0,y) = w(a,y) = 0 \;, \qquad (5.3) \\
&\psi_x(x,0) = \psi_x(x,b) = 0 \;,\; M_2(x,0) = M_2(x,b) = 0 \;, \\
&\psi_y(0,y) = \psi_y(a,y) = 0 \;,\; M_1(0,y) = M_1(a,y) = 0 \;.
\end{aligned}$$

Again, the finite-element solution (obtained by using 2 x 2 mesh of eight-node quadratic elements with reduced integration: 2Q8-R) is in close agreement with the closed-form solution.

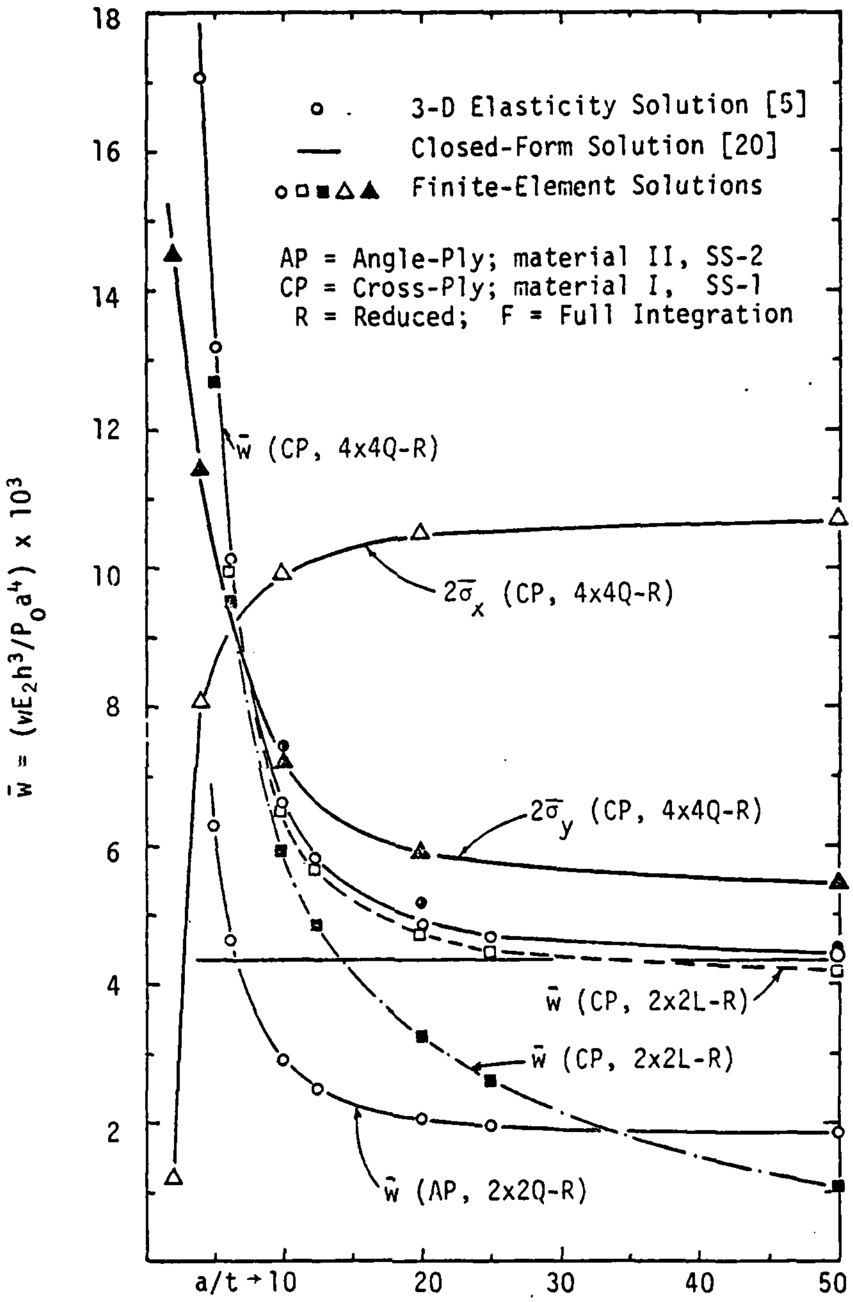

Figure 1. Comparison of the exact closed-form solution and finite-element solution for four-layer (0°/90°/90°/0°), 45°/-45°/45°/45°) square plates under sinusoidal loading.

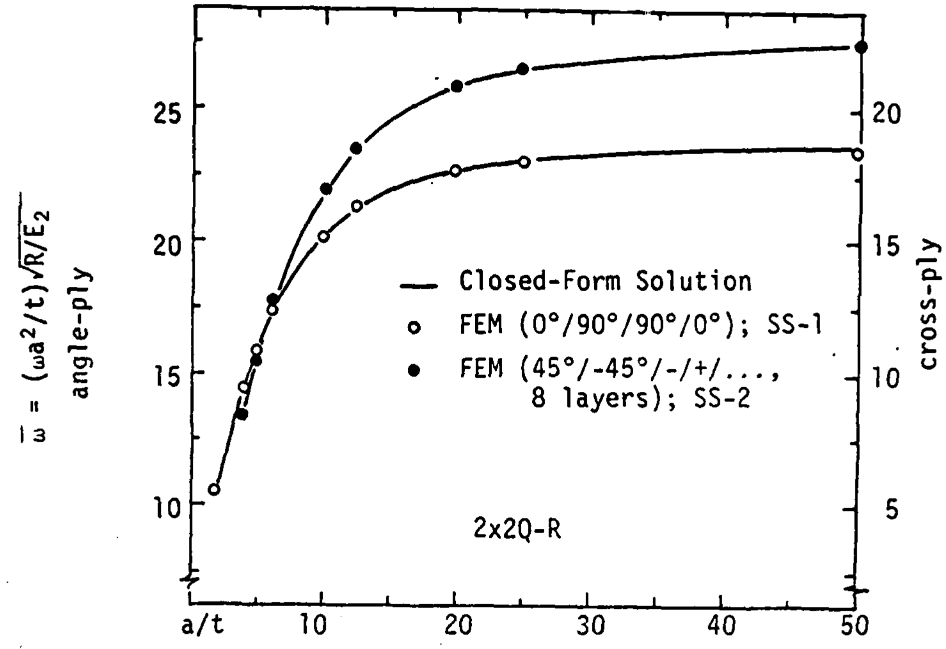

Figure 2. Comparison of the closed-form and finite element solution for nondimensionalized fundamental frequencies square plates (material II).

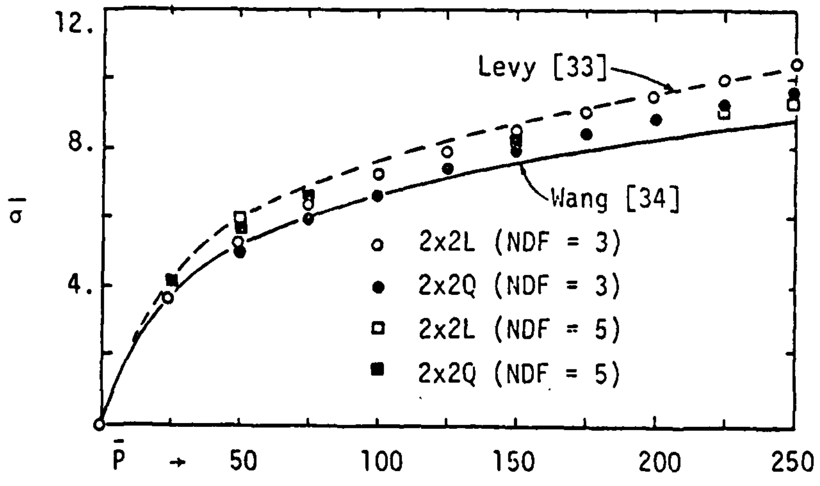

Figure 3. Comparison of the nondimensionalized stress for simply supported (SS-3), isotropic (ν = 0.3) square plate under uniform loading.

Figure 2 shows the nondimensionalized fundamental frequency for four-layer, cross-ply (0°/90°/90°/0°), and eight-layer, angle-ply (45°/-45°/+ /- ...) square plates of material II. The support conditions for the cross-ply plate were assumed to be those in SS-1, and the support conditions used for angle-ply plate were those in SS-2. Results for both of the cases were obtained using mesh 2Q8-R. The finite-element solutions are gratifyingly close to the exact closed-form solutions.

Having established the credibility of the finite element developed herein for the linear analysis of layered composite plates, we now employ the element in the nonlinear analyses. First, results are presented for single-layer isotropic square plate under uniform loading. The essential boundary conditions used are:

$$
\begin{aligned}
&\text{simply-supported (SS-3):} && u = v = w = 0 \text{ on all edges.} \\
&\text{clamped (CC-1):} && u = v = w = 0 \text{ on all edges,} \\
& && \psi_x = 0 \text{ along edges parallel to x-axis,} \\
& && \psi_y = 0 \text{ along edges parallel to y-axis.}
\end{aligned}
\qquad (5.4)
$$

Figures 3 to 5 show the nondimensionalized deflection, $\bar{w} = w/h$, and nondimensionalized stress, $\bar{\sigma} = \sigma a^2/Eh^2$, as a function of the load parameter, $\bar{P} = P_0 a^4/Eh^4$ for clamped (CC-1) square plate, and simply-supported (SS-3) square plate, respectively. The results are compared with the Ritz solution of Way [32], double Fourier-series solution of Levy [33], the finite-difference solution of Wang [34], the Galerkin solution of Yamaki [35], and the displacement finite-element solution of Kawai and Yoshimura [36]. Finite-element solutions were computed for the five degrees of freedom (NDF = 5), and for three degrees of freedom (NDF = 3); in the latter case, the in-plane displacements were suppressed. Tbe present solutions are in good agreement with the results of other investigators. Since suppressing the

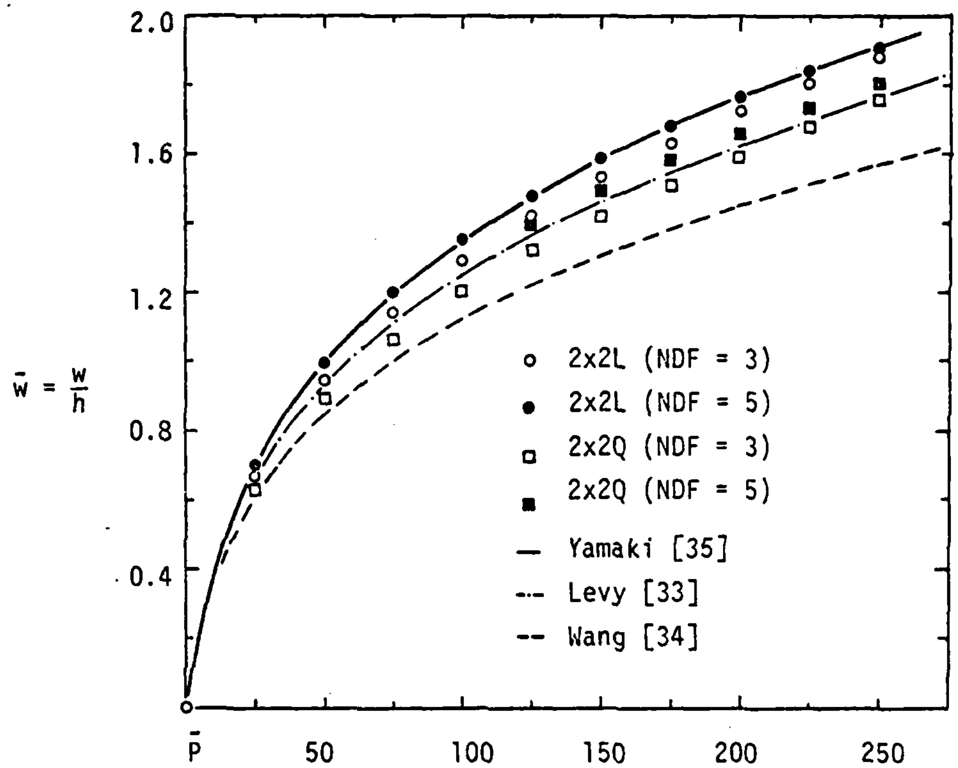

Figure 4. Comparison of the nondimensionalized deflection for simply-supported (SS-3), isotropic (ν = 0.3) square plate under uniformly distributed pressure load.

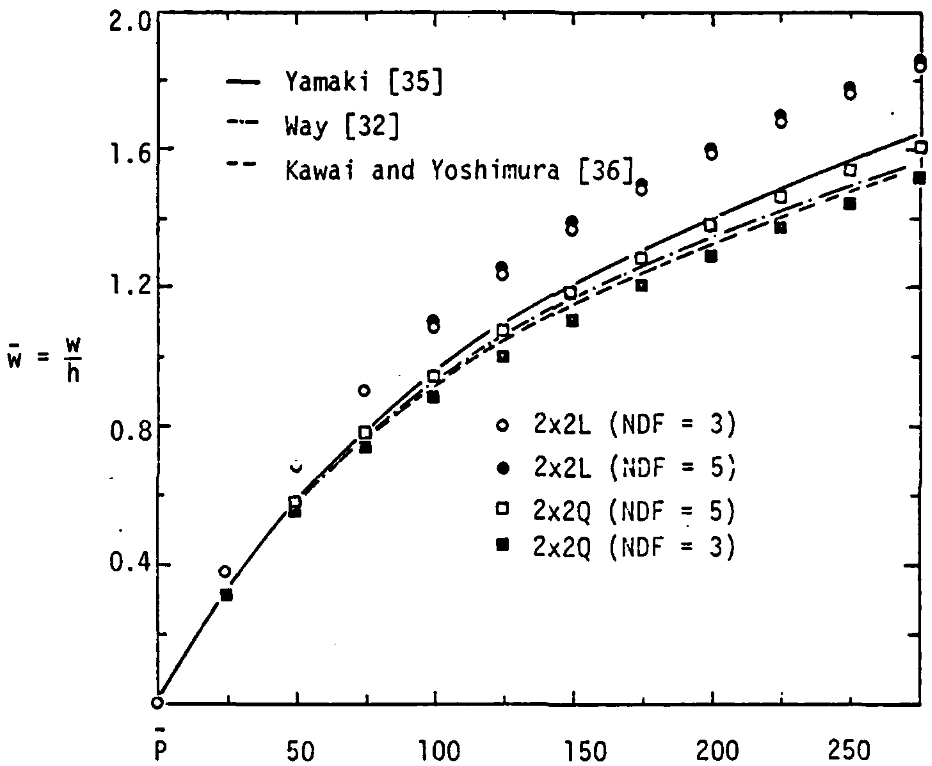

Figure 5. Comparison of the nondimensionalized deflection for clamped (CC-1), isotropic (ν = 0.3) square plate under uniformly distributed pressure load.

in-plane displacements stiffens the plate, the deflections are smaller and stresses are larger than those obtained by including the in-plane displacements. Solutions of the other investigators were read from the graphs presented in their papers.

Next, results of the large-deflection bending analysis of layered composite, thin ($a/h = 40$) plates are presented. Figure 6 shows the non-dimensionalized deflection versus the load parameter for two-, and six-layer, anti-symmetric (0°/90°/0°/...) cross-ply rectangular plates of material II, subjected to uniform loading. The plate is assumed to be clamped (CC-2) in the following sense:

$$\begin{aligned} w &= \psi_x = 0 \text{ along edges parallel to y-axis,} \\ w &= \psi_y = 0 \text{ along edges parallel to x-axis.} \end{aligned} \tag{5.5}$$

The present solution is in good agreement, for various aspect ratios, with the perturbation solution of Chia and Prabhakara [27]. Due to lack of tabulated results in [27], the relative differences in the two solutions cannot be discussed. It is clear that the nonlinear load-deflection curve is not deviated so much from the linear load-deflection line.

Figure 7 shows similar results for two-, and six-layer, angle-ply (45°/-45°/-/+...), and clamped (CC-2) rectangular plate (material II) subjected to uniform loading. Again, the present result is in close agreement with that of Chia and Prabhakara [28]. The nondimensionalized stress, $\bar{\sigma}_x$, for the cross-ply and angle-ply plates discussed above is plotted against the load parameter in Figure 8.

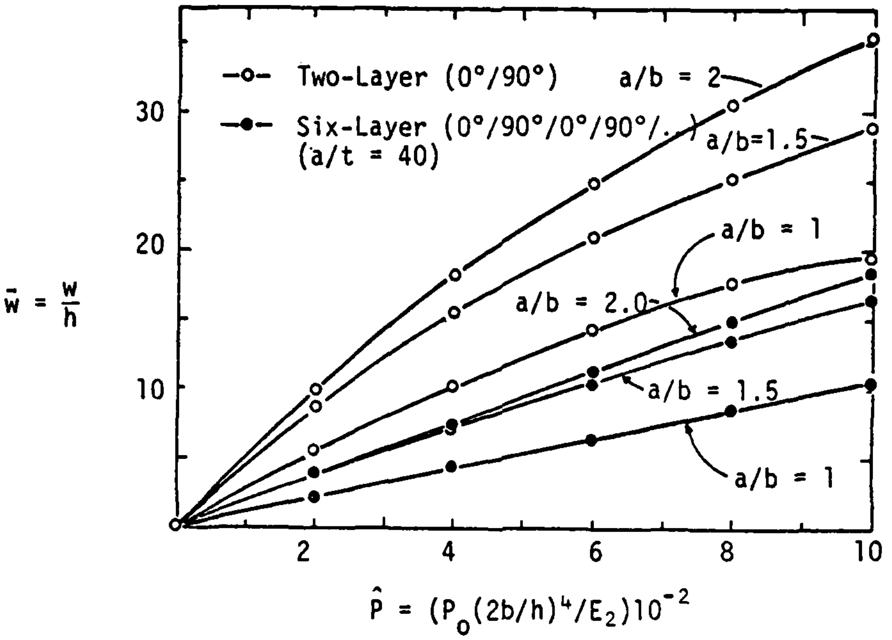

Figure 6. Load-deflection curves for antisymmetric cross-ply clamped (CC-2), rectangular plates (material II) under uniform loading

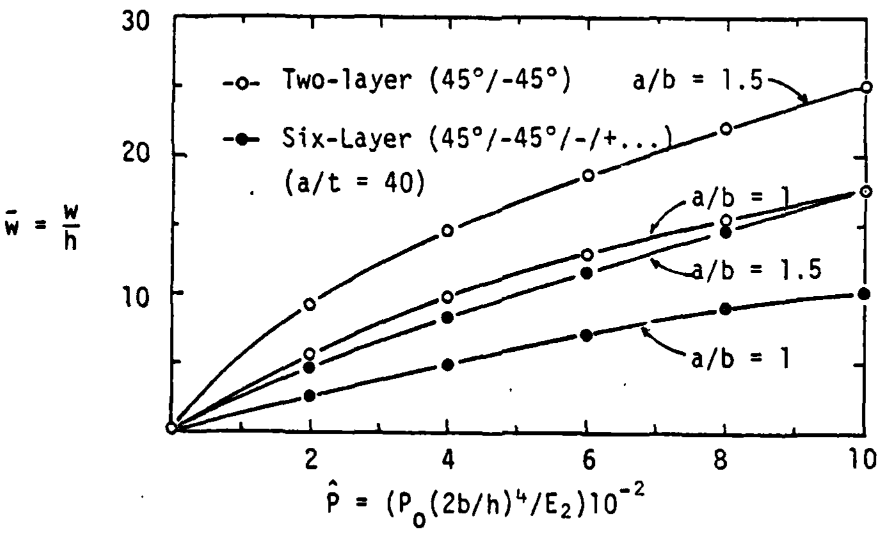

Figure 7. Load-deflection curves for antisymmetric angle-ply clamped (CC-2), rectangular plates (material II) under uniform loading.

The effect of the transverse shear strain on the deflection and stresses on the load-deflection, and load-stress curves is shown in Fig. 9. Note that the deflection for a/h = 10 is about 30% larger than that for a/h = 100, at $\hat{P} = 10$. That is, the deflections predicted by the classical thin-plate theory are lower than those predicted by the shear deformable theory.

Figure 10a shows the ratio of nonlinear to linear fundamental frequencies versus the amplitude-to-thickness ratio for two-layer angle-ply ($\theta/-\theta$), clamped (CC-2) square plate of material II. The side-to-thickness ratio (a/h) was taken to be 40 (i.e., thin plate). Similar results are presented in Fig. 10b for two-layer, cross-ply (0°/90°), thick rectangular plates of material II. The boundary conditions used were simply-supported (SS-1), and clamped (CC-3):

$$\begin{aligned} u = w = \psi_x = 0 &\quad \text{along edges parallel to y-axis} \quad , \\ v = w = \psi_y = 0 &\quad \text{along edges parallel to x-axis} \quad . \end{aligned} \tag{5.6}$$

The side-to-thickness ratio used in this case was, b/h = 10 (i.e., thick plate). Since the present boundary conditions are somewhat different from those used by Chia and Prabhakara [28], the present solutions do not coincide with those in [28].

6. SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

A finite-element model is developed based on the combined theory of Yang, Norris, and Stavsky [16] and von Karman. That is, the model accounts for the transverse shear strain, and large rotations. Numerical results are presented for linear and nonlinear deflections, stresses, and natural frequencies of rectangular plates subjected to various edge conditions. The finite-element solutions are compared with the exact closed-form solutions

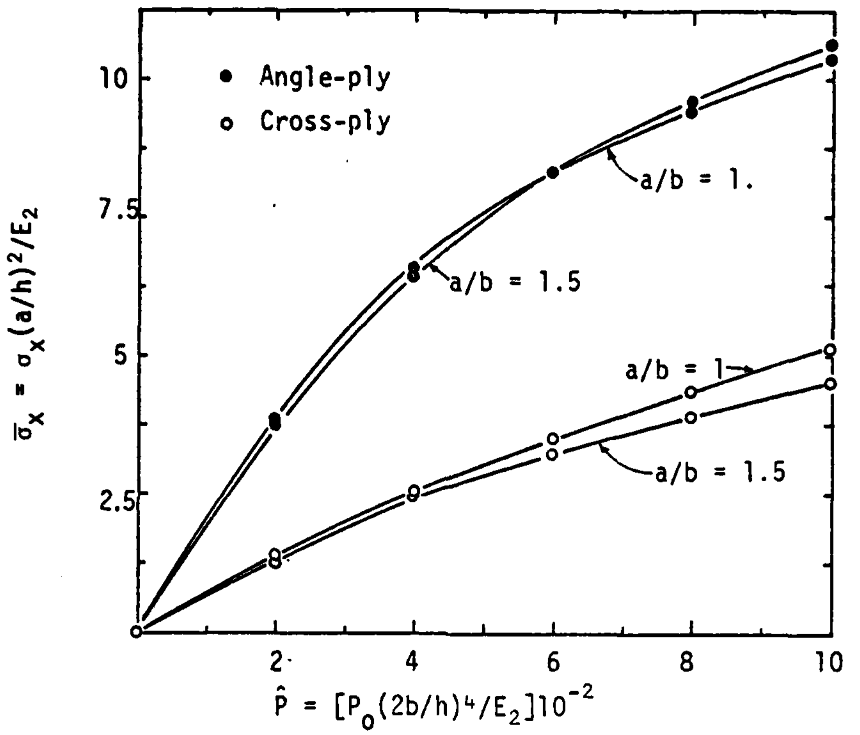

Figure 8. Load-stress curves for two-layer clamped (CC-2), rectangular plates (material II) under uniform loading.

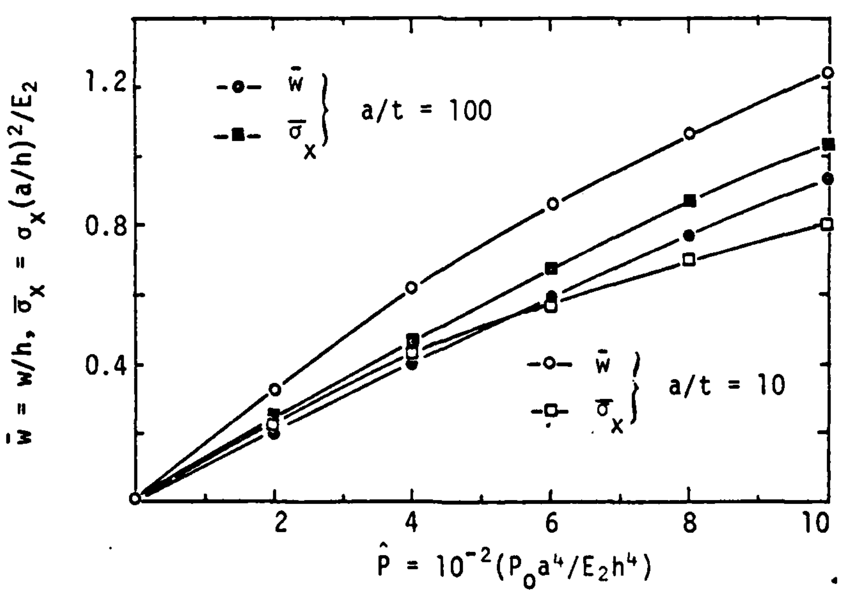

Figure 9. Effect of the transverse shear on the load-deflection and load-stress curves for four-layer (0°/90°/90°/0°) simply-supported (SS-1) square plate (material I) under uniform loading.

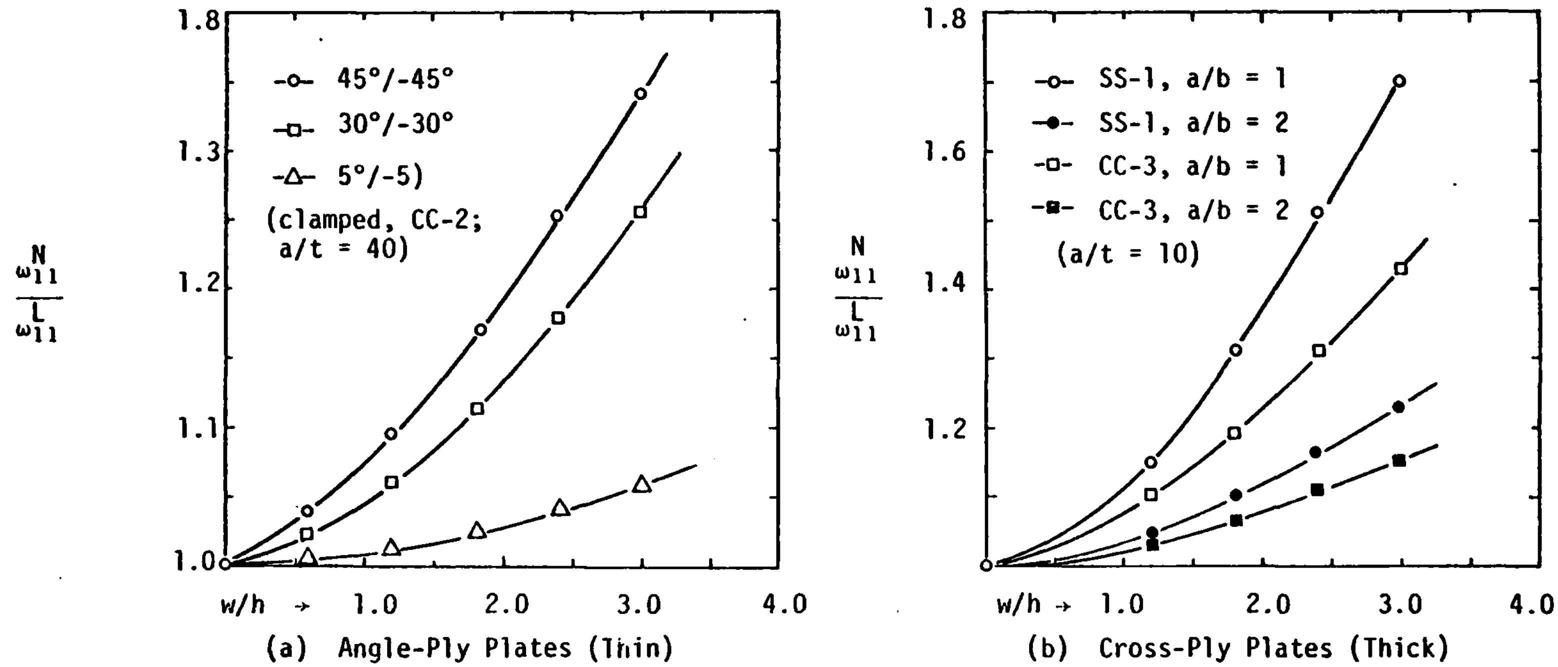

Figure 10. Ratio of nonlinear frequency to linear frequency as a function of the ratio of amplitude to thickness for square plates (material II).

in the linear case, and with the perturbation solution in the nonlinear case.

The finite-element solutions are found to be in excellent agreement with the exact closed-form solutions in the linear analysis. In the nonlinear analysis, the finite-element solutions are in fair agreement with the perturbation solution; of course, there is no proof that the perturbation solution is close to the exact. The load-deflection curve in the shear deformable theory does not deviate much from the linear theory, when compared to the load-deflection curve in the von Karman theory.

The finite-element developed herein is algebraically simple, and involves fewer degrees of freedom per element compared to traditional finite elements. Application of the present element (or an element based on the combined theory) to the following problem areas, at this writing, is either in development or awaiting:

- Transient analysis of layered composite plates (linear as well as nonlinear)
- Transient analysis of bimodulus (see [37-40]) composite plates (linear and nonlinear)
- Forced vibration of ordinary and bimodulus composite plates
- Static and transient (linear and nonlinear) analysis of plates with cut-outs
- All of the above for cylindrical and doubly-curved thick shells [41].

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REFERENCES

1. E. Reissner and Y. Stavsky, 'Bending and Stretching of Certain Types of Heterogeneous Aeolotropic Elastic Plates', J. of Appl. Mech., 28, 402-408 (1961).

2. S. B. Dong, K. S. Pister, and R. L. Taylor, 'On the Theory of Laminated Anisotropic Shells and Plates', J. Aerospace Sciences, 29, 969-975 (1962).

3. C. W. Bert and B. L. Mayberry, 'Free Vibrations of Unsymmetrically Laminated Anisotropic Plate with Clamped Edges', J. Composite Materials, 3, 282-293 (1969).

4. N. J. Pagano, 'Exact Solutions for Composite Laminates in Cylindrical Bending', J. of Composite Materials, 3(3), 398-411 (1969).

5. N. J. Pagano and S. J. Hatfield, 'Elastic Behavior of Multilayer Bi-directional Composites', AIAA Journal, 10, 931-933 (1972).

6. S. Srinivas and A. K. Rao, 'Bending, Vibration, and Buckling of Simply Supported Thick Orthotropic Rectangular Plates and Laminates', Int. J. Solids and Structures, 6, 1463-1481 (1970).

7. C. W. Pryor Jr. and R. M. Barker, 'A Finite Element Analysis Including Transverse Shear Effects for Applications to Laminated Plates', AIAA J. 9, 912-917 (1971).

8. R. M. Barker, F. T. Lin, and J. R. Dara, 'Three-Dimensional Finite-Element Analysis of Laminated Composites', National Symposium on Computerized Structural Analysis and Design, George Washington University (1972).

9. P. K. Sinha and A. K. Rath, 'Vibration and Buckling of Cross-Ply Laminated Circular Cylindrical Panels', Aeronautical Quarterly, 26, 211-218 (1975).

10. C. W. Bert and T. L. C. Chen, 'Effect of Shear Deformation on Vibration of Antisymmetric Angle-Ply Laminated Rectangular Plates', Int. J. Solids and Structures, 14, 465-473 (1978).

11. A. S. Mawenya and J. D. Davies, 'Finite Element Bending Analysis of Multilayer Plates', Int. J. Num. Meth. Engng., 8, 215-225 (1974).

12. S. C. Panda and R. Natarajan, 'Finite Enement Analysis of Laminated Composite Plates', Int. J. Num. Meth. Eegng., 14, 69-79 (1979).

13. R. L. Spilker, S. C. Chou and O. Orringer, 'Alternate Hybrid Stress Elements for Analysis of Multilayer Composite Plates', J. Composite Materials, 11, 51-70 (1977).

14. J. N. Reddy, 'Free Vibration of Antisymmetric, Angle-Ply Laminated Plates, Including Transverse Shear Deformation by the Finite Element Method', J. Sound Vib., 66(4), 565-576 (1979).

15. J. N. Reddy, 'A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates', Int. J. Numer. Meth. Engng. (1980), in press.

16. P. C. Yang, C. H. Norris, and Y. Stavsky, 'Elastic Wave Propagation in Heterogeneous Plates', Int. J. Solids and Structures, 2, 665-684 (1966).

17. J. M. Whitney and N. J. Pagano, 'Shear Deformation in Heterogeneous Anisotropic Plates', J. Appl. Mech., 37, 1031-1036 (1970).

18. J. N. Reddy, 'Simple Finite Elements with Relaxed Continuity for Nonlinear Analysis of Plates', Finite Element Methods in Engineering (Proc. Third Int. Conf. in Australia on Finite Element Methods), A. P. Kabaila and V. A. Pulmano (eds.), The University of New South Wales, Sydney, Australia, 265-281 (1979).

19. R. Courant, Calculus of Variations and Supplementary Notes and Exercises, revised and amended by J. Moser, New York University (1956).

20. J. N. Reddy, 'A Comparison of Closed-Form and Finite Element Solutions of Thick, Laminated, Anisotropic Rectangular Plates', Research Report No. OU-AMNE-79-19, School of Aerospace, Mechanical and Nuclear Engineering, University of Oklahoma, Norman (1979).

21. J. M. Whitney and A. W. Leissa, 'Analysis of Heterogeneous Anisotropic Plates', J. Appl. Mech., 36, 261-266 (1969).

22. J. A. Bennett, 'Nonlinear Vibration of Simply Supported Angle Ply Laminated Plates', AIAA J., 9, 1997-2003 (1971).

23. C. W. Bert, 'Nonlinear Vibration of a Rectangular Plate Arbitrarily Laminated of Anisotropic Material', J. Appl. Mech., 40, 452-458 (1973).

24. R. Chandra, and B. B. Raju, 'Large Amplitude Flexural Vibration of Cross Ply Laminated Composite Plates', Fibre Science and Technology, 8, 243-263 (1975).

25. R. Chandra, 'Large Deflection Vibration of Cross-Ply Laminated Plates with Certain Edge Conditions', J. Sound and Vibration, 47(4), 509-514 (1976).

26. S. A. Zaghloul and J. B. Kennedy, 'Nonlinear Analysis of Unsymmetrically Laminated Plates', J. Engng. Mech. Div., ASCE, 101(EM3), 169-185 (1975).

27. C. Y. Chia and M. K. Prabhakara, 'Large Deflection of Unsymmetric Cross-Ply and Angle-Ply Plates', J. Mech. Engng. Sci., 18(4), 179-183 (1976).

28. C. Y. Chia and M. K. Prabhakara, 'A General Mode Approach to Nonlinear Flexural Vibrations of Laminated Rectangular Plates', J. Appl. Mech., 45, 623-628 (1978).

29. A. K. Noor and S. J. Hartley, 'Effect of Shear Deformation and Anisotropy on the Non-Linear Response of Composite Plates', Developments in Composite Materials- 1, G. Holister (ed.), Applied Science Publishers, Barking, Essex, England, 55-65 (1977).

30. O. C. Zienkiewicz, R. L. Taylor, and J. M. Too, 'Reduced Integration Technique in General Analysis of Plates and Shells', Int. J. Num. Meth. Engng., 3, 575-586 (1971).

31. J. M. Whitney, 'Stress Analysis of Thick Laminated Composite and Sandwich Plates', J. Composite Materials, 6, 426-440 (1972).

32. S. Way, 'Uniformly Loaded, Clamped, Rectangular Plates with Large Deformation', Proc. 5th Int. Congr. Appl. Mech. (Cambridge, Mass., 1938), John Wiley, 123-128 (1939).

33. S. Levy, 'Bending of Rectangular Plates with Large Deflections', Report No. 737, NACA (1942).

34. C. T. Wang, 'Bending of Rectangular Plates with Large Deflections', Report No. 1462, NACA (1948).

35. N. Yamaki, 'Influence of Large Amplitudes on Flexural Vibrations of Elastic Plates', ZAMM, 41, 501-510 (1967).

36. T. Kawai and N. Yoshimura, 'Analysis of Large Deflection of Plates by the Finite Element Method', Int. J. Numer. Meth. Engng., 1, 123-133 (1969).

37. C. W. Bert, 'Recent Advances in Mathematical Modeling of the Mechanics of Bimodulus, Fiber-Reinforced Composite Materials', Recent Advances in Engineering Science (Proc. 15th Annual Meeting of the Society of Engineering Science), R. L. Sierakowski (ed.), University of Florida, Gainesville, FL, 101-106 (1978).

38. J. N. Reddy and C. W. Bert, 'Analysis of Plates Constructed of Fiber-Reinforced Bimodulus Composite Material', Mechanics of Bimodulus Materials, C. W. Bert (Ed.), AMD-Vol. 33, The American Society of Mechanical Engineers, New York, 67-83 (1979).

39. J. N. Reddy and W. C. Chao, 'Finite-Element Analysis of Laminated Bimodulus Composite-Material Plates', Computers and Structures (1980), in press.

40. C. W. Bert, J. N. Reddy, V. Sudhakar Reddy, and W. C. Chao, 'Analysis of Thick Rectangular Plates Laminated of Bimodulus Composite Materials', AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics and Maetrials Conference, Seattle, Washington, May 12-14 (1980).

41. C. W. Bert and V. Sudhakar Reddy, 'Cylindrical Shells of Bimodulus Composite Material', ASCE National Convention, Hollywood, Florida, October 27-31 (1980).

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Finite-element analysis of the large-deflection theory (in von Karman's sense), including transverse shear, governing moderately thick, laminated anisotropic composite plates is presented. Linear and quadratic rectangular elements with five degrees of freedom (three displacements, and two shear rotations) per node are employed to analyze rectangular plates subjected to various loadings and edge conditions. Numerical results for bending deflections, stresses, and natural frequencies are presented showing the parametric effects of plate aspect ratio, side-to-thickness ratio, orientation of layers, and anisotropy.

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